

2018 Courses

Overview

Legend:

(H) - Hands-on class where you'll be making things as part of learning

(P) - Pencil and paper work will be an important part of learning in this class

(L) - Lecture-style class

(C) - Computer-based class, laptop needed

(E) - Exploratory class where students are offered problems and staff support, but are expected to self-direct their work.

(*) - Course with prerequisites. This course is challenging, if in doubt, talk to us about it.

Lower-case characters indicate that a feature is present in a class but not primary.

- **Geometry of Curvature** (HP)
- **Number Sense** (HP)
- **The Cosmic Distance Ladder** (PLh)
- **Spirals** (HCe)
- **Iteration** (PLC)
- **Language Games** (P)
- **Recurrences** (PL)
- **Finite Fields** (PL)
- **Analytic Combinatorics** (*PL, Prereq: Calculus)
- **Discrete Dynamical Systems** (*PLC, Prereq: Calculus)
- **Statistical Simulation with R** (*CE)
- **Euclid in Greek** (PL)
- **Newton in Latin** (PL)

For first-time campers the courses “Geometry of Curvature”, “Number Sense”, and “Iteration” are highly recommended. For those who want to put pencil to paper and start solving hard problems, “Recurrences”, “Finite Fields”, and “Language Games” will keep your gears turning.

While some courses are advanced and have prerequisites, all courses are fun and no course is “easy”! In every course we have the ability to take students as deeply into advanced mathematics as they want to go.

Detailed Descriptions

Geometry of Curvature

Instructor: Rolfe Schmidt

Starting with some Geometry-by-experiment we learn the need for proof and axioms and build our own simple axiom systems. Not satisfied, we turn to Euclid and try our hands at a few proofs to see the power of this system that stood the test of time for 2000 years. But are we satisfied? Of course not. We turn back to experimental geometry and construct models that “break” Euclidean geometry. By understanding why Euclid’s rules did not apply we will learn the significance of each of Euclid’s axioms at a deep level and open the door to the geometry of curved surfaces and the remarkable properties of symmetric spaces.

Number Sense

Instructor: Wendy K. Tam Cho

Fractals, Pascal’s Triangle, figurate numbers... What seems like simple arithmetic can suddenly give rise to all sorts of fun. In this class we will put pencil (and pen and marker and scissors and tape) to paper and see first hand what structures arithmetic can produce and how it happens. Once we see patterns and structure, we will start learning how to formalize our observations and move from conjecture to proof.

The Cosmic Distance Ladder

Instructors: Trevor Layman and Rolfe Schmidt

Eratosthenes was able to measure the circumference of the earth. Aristarchus built on this to measure the distance to the moon, and even to the sun. He realized the sun was enormous and very far away, and concluded Earth can’t be the center of the universe. Knowing the size of the earth’s orbit, Bessel was able to use parallax to measure the distance to nearby stars. These stars were seen to have an interesting characteristic: their absolute brightness mostly depended on their color - this lets us measure the distance to much more distant stars and even measure the size of our galaxy. The progression continues until we measure the size of the observable universe. Each of these steps builds on the previous one, and lets us climb a bit further up this “cosmic distance ladder”. We’ll climb as far as we can, learning subtle geometry, algebra, and perhaps some relativity. We’ll also touch on primary sources and learn about the history of thought over the last 2500 years.

Spirals

Instructor: Rolfe Schmidt

You may have heard about how the spirals in a sunflower (or pineapple or artichoke or...) come in Fibonacci-numbered sets. If so, you've probably heard that this has something to do with the Golden Ratio. Which then has something to do with Greek Architecture and the distance between your belly button and our head. It all sounds magical, eh?

In this class you'll either learn to be a magician or you'll learn to discern magic from truth, but we can't tell you which. We will see how natural approximations of algebraic numbers (like the Golden Ratio) lead to recurrences (like the Fibonacci numbers). We'll use this to see why simple dynamical systems driven by algebraic equations will always produce "spirals", and they don't need to come in Fibonacci-numbered sets.

Not only that, we will play with JavaScript programs to make pictures of the objects we are studying. This will open up several new cans of worms: to get nice pictures we will need to learn about Voronoi diagrams and cubic splines.

This is a hands-on class that will offer a few launching points for students to take off in their own direction, whether it is theoretical, computational, artistic, or all three.

Iteration

Instructor: Burton Newman

Enter a number in your calculator. Now press the square button repeatedly and watch the numbers evolve on the screen. Can you predict the long-term behavior of this sequence? Will the numbers get larger and larger, or does something else happen? What happens if you change the starting number? If these questions are too easy, are there any other operations you can try to perform repeatedly? In this course we will study discrete dynamical systems. Instead of calculating on paper, we will use Python to perform numerical experiments, which will allow us to quickly formulate and test our conjectures. We will then use Desmos to gain insight into the causes of the behavior we are observing.

Language Games

Instructor: Trevor Layman

Can you decode a Hittite tablet? Or piece together the meaning of a Quechua phrase from a few known fragments? How on earth could people figure out conclusively that Linear-B was used to write a previously unknown dialect of ancient Greek? In this class we will explore these sorts of language problems. We'll see that even though they often seem impossible to start,

through systematic and careful reasoning - in other words with logic - we can unlock new knowledge with confidence that intuition will never provide.

We will specifically work on problems from the North American Computational Linguistics Olympiad (NACLO), a nationwide competition combining linguistic and mathematical components. In this course, Trevor, who has prepared both middle and high school students for NACLO, will present past exams and work through practice problems in an engaging way. By the end of the course, students can expect to feel comfortable with the structure of NACLO and the types of problems presented, should they wish to participate in NACLO 2019.

Recurrences

Instructor: Christian Geske

How many moves does it take to solve the Towers of Hanoi puzzle with 64 disks? How many regions can you get when you cut a circle with 100 straight lines? How many 30-bit bit strings are there with no neighboring "1"s? How do you pick a seat for the [Josephus Problem](#)? All of these problems have similar solutions: we can solve the problem by brute force for small numbers, then we can find a rule that lets us write a solution for a bigger number in terms of smaller ones. These rules are called recurrences and they are a powerful, general, and fundamental part of discrete mathematics.

In this class we will combine individual and collaborative problem solving to uncover general principles in the mathematics of recurrences.

Finite Fields

Instructor: Burton Newman

You may have spent many years working with number systems like the rational numbers (e.g. $3/4$), the real numbers (e.g. 3.1415926...) and the complex numbers (e.g. $3 + 4i$). In each of these number systems you can add, subtract, multiply and divide. But there are actually many other number systems out there in mathematical nature with these four operations. And as we can carry out the same kinds of operations in these systems, we can also ask the same kinds of questions. Unlike exotic systems such as the real numbers, many of the systems we study can be stored entirely on a computer - no approximations needed. These systems might seem like little novelties, but they exhibit patterns that have fascinated mathematicians for centuries, and have real-world applications as well. In this course we will gain hands-on experience performing computations in such systems. Just as we do over the real numbers, we will factor polynomials and solve equations over such systems. And we will see applications of the theory we develop to geometry, combinatorics and error-correcting code

Analytic Combinatorics

Instructor: Christian Geske

An active area of computer science and discrete math research is centered around the remarkable observation that we can use the idea of the *generating function* to transform many combinatorics problems into questions about algebra. For some problems - like Sicherman's Dice - we can turn a question about counting into a question about polynomials and factorization. Where things get really exciting, though, is where we take on infinite combinatorial classes and leave the world of polynomials for power series on the complex plane. In this class we'll learn a powerful toolkit that allows you to solve large classes of combinatorial problems.

Prerequisites: *Calculus, particularly convergence and manipulation of power series. Combinatorics at the level of AoPS Intermediate Counting and Probability.*

Discrete Dynamical Systems

Instructor: Burton Newman

Iteration is a fundamental act of computation: take some operation and repeat it again and again and again. The action is *discrete* (it happens in "lumps", not continuously) and it is *dynamic* (it changes with each step) so we call it a *discrete dynamical system*. These sound simple, but quickly lead to surprising, deep, and difficult behavior. Unsolved and unsolvable problems abound. In this *calculus-based* version of the *Iteration* class, we will use both computer models and theory to explore the behavior of discrete dynamical systems.

Prerequisites: *Calculus. Programming experience helpful but not required.*

Statistical Simulation with R

Instructor: Wendy K. Tam Cho

Very few real-world problems have tidy, closed-form solutions. With numerical methods and approximation theorems in hand, you won't need to let this slow you down. In this class we will learn to use R Studio to model systems and analyze their statistical properties.

Euclid in Greek

Instructor: Trevor Layman

In this course, we will explore the work of Euclid, the "father of geometry," in its original language. First, we will study the Greek alphabet, and then we will use it to unlock some of

Euclid's famous axioms. Classical scholar Trevor Layman will share his insights and expertise with students as they follow Euclid's line of reasoning.

Newton in Latin

Instructor: Trevor Layman

Isaac Newton's Principia is considered one of the most important works in the history of science. In this course, classical scholar Trevor Layman will highlight the most famous parts of the text, in its original language- Latin. No prior knowledge of the Latin language is expected, instead, students will deduce meaning with the help of English cognates and helpful notes.